IASSNS-HEP-00/35
NSF-ITP-00-36
PUPT-1927

Matrix Theory Interpretation of DLCQ String Worldsheets

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Abstract

We study the null compactification of type-IIA-string perturbation theory at finite temperature. We prove a theorem about Riemann surfaces establishing that the moduli spaces of infinite-momentum-frame superstring worldsheets are identical to those of branched-cover instantons in the matrix-string model conjectured to describe M-theory. This means that the identification of string degrees of freedom in the matrix model proposed by Dijkgraaf, Verlinde and Verlinde is correct and that its natural generalization produces the moduli space of Riemann surfaces at all orders in the genus expansion.

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[2]Supported by PSC-CUNY Research Award Program Grants nos. 668460, 61508-00, 69466-00, a CUNY Collaborative Incentive grant 91915-00-06 and the National Science Foundation under Grant No. PHY94-07194. Permanent address: Center for Theoretical Physics, The Graduate School and University Center and Baruch College, The City University of New York, New York, NY.

[3]Supported by NSF grant PHY98-02484 and NSERC of Canada.

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It is widely believed that each of the five known consistent string theories are limits of a single eleven-dimensional theory called M-theory. While this theory has not yet been fully mathematically formulated, there is an interesting proposal, namely the *matrix model* [1, 2]. This model is conjectured to describe M-theory in a particular kinematical region, the infinite-momentum frame. In the matrix model, the superstring is a composite object resembling a necklace of D0-branes.

A nontrivial check of the matrix model proposal would be to use it to obtain perturbative string theory. There are already convincing arguments that the non-interacting type-IIA strings emerge in the appropriate limit of the matrix model [3]. There are several approaches to deriving superstring interactions along these lines [3]-[9]. In these, matrix-theory instantons interpolate between initial states and final states of strings through a Riemann surface which is a branched cover of the cylinder. Whether these branched-cover instantons account correctly for string-scattering amplitudes is an important question.

In this Letter, we show for the first time that in the appropriate context, perturbative string theory can be formulated using *only* branched-cover Riemann surfaces.

The matrix model is a formulation of M-theory in the infinite-momentum frame. Compactification of a spatial direction of M-theory produces type-IIA string theory. The matrix model becomes 1+1-dimensional, maximally-supersymmetric Yang-Mills theory.

To compare the matrix model directly with infinite-momentum-frame string theory, we study the string path integral with a compactified null direction. The natural quantization of the string in this frame is discrete light-cone quantization (DLCQ). We find it necessary to introduce a finite temperature by further compactifying Euclidean time. We prove a new theorem on Riemann surfaces, stating that to any order in the genus expansion, these compactifications restrict the worldsheets to branched covers of a torus. This differs from the moduli space of strings without these compactifications, which includes *all* Riemann surfaces up to conformal diffeomorphisms.

The same branched covers appear in the matrix model. According to Dijkgraaf, Verlinde and Verlinde [3], the string degrees of freedom are simultaneous eigenvalues of the matrices. At finite temperature, the matrices are defined on a torus [10] and their eigenvalues, since they solve polynomial equations, are functions on branched covers of the torus. If the matrix model is to agree with perturbative string theory, these branched covers must be the full set of Riemann surfaces that contribute to the string path integral.

We compare only the degrees of freedom of the two theories and not the energy spectra. However, to one-loop order, the spectra are known to coincide [10]. The coincidence of energy spectra once higher order corrections are included is still an open question.

The string path integral for the vacuum energy is

$$F = -\sum_{g,\sigma} g_s^{2g-2} \int [dh_g dX d\Psi] \exp\left(-\frac{1}{4\pi\alpha'}\int \sqrt{h}\left(h^{ab}\partial_a X^\mu \partial_b X^\mu - 2\pi i\alpha' \Psi^\mu \gamma\cdot\nabla\Psi^\mu\right)\right) \tag{1}$$

Here, we use, for example, the Neveu-Schwarz-Ramond superstring[5]. The string coupling constant is g_s and its powers weight the genus, $g = 0, 1, \ldots$, of the string's worldsheet. There is also a sum over spin structures, σ which, with the appropriate weights, imposes the GSO projection. For each value of the genus, g, $[dh_g]$ is an integration measure over all metrics of that genus and is normalized by dividing out the volume of the worldsheet re-parameterization and Weyl groups. We will assume that the metrics of both the worldsheet and the target spacetime have Euclidean signatures.

We wish to study the situation where the target space has particular compact dimensions. Two compactifications will be needed. The first compactifies the light-cone in Minkowski space by making the identification $\frac{1}{\sqrt{2}}(t - x^9) \sim \frac{1}{\sqrt{2}}(t - x^9) + 2\pi R$. In our Euclidean coordinates it is the identification

$$\left(X^0, \vec{X}, X^9\right) \sim \left(X^0 + \sqrt{2}\pi i R, \vec{X}, X^9 - \sqrt{2}\pi R\right) \tag{2}$$

With this compactification the GSO projection is unmodified. The factor of i in the identification of X^0 might seem unnatural since it identifies a real integration variable periodically in a complex direction. However, we shall see that, for the path integral at genus 1, where we can check the result independently by using operator methods to compute the same partition function, this identification is indeed the correct thing to do. We shall postulate that it also gives the correct partition function at genus greater than one.

The second compactification that we shall need is that of Euclidean time,

$$\left(X^0, \vec{X}, X^9\right) \sim \left(X^0 + \beta, \vec{X}, X^9\right) \tag{3}$$

This compactification, with the appropriate modification of the GSO projection to make space-time fermions anti-periodic, introduces a temperature, $T = 1/k_B\beta$ where k_B is Boltzmann's constant, so that (1) computes the thermodynamic free energy.

In order to implement this compactification in the path integral, we assume that the worldsheet is a Riemann surface Σ_g of genus g whose homology group $H_1(\Sigma_g)$ is generated by the closed curves,

$$\begin{aligned} &a_1, a_2, \ldots, a_g \ , \ b_1, b_2, \ldots, b_g \\ &a_i \cap a_j = \emptyset \ , \ b_i \cap b_j = \emptyset \ , \ a_i \cap b_j = \delta_{ij} \end{aligned} \tag{4}$$

[5]Most of our considerations apply to the bosonic sector of any string theory. We use the superstring as an example. For the relationship with matrix theory, however, supersymmetry is important and that case is more closely related to the Green-Schwarz superstring.

Furthermore, one may pick a basis of holomorphic differentials $\omega_i \in H^1(\Sigma_g)$ with the properties

$$\oint_{a_i} \omega_j = \delta_{ij} \quad , \quad \oint_{b_i} \omega_j = \Omega_{ij} \tag{5}$$

where Ω is the period matrix. It is complex, symmetric, $\Omega_{ij} = \Omega_{ji}$, and has positive definite imaginary part.

Compactification is implemented by including the possible windings of the string worldsheet on the compact dimensions. These form distinct topological sectors in the path integration in (1). In the winding sectors, the bosonic coordinates of the string should have a multi-valued part which changes by β·integer or $(i)\sqrt{2}R$·integer as it is moved along a homology cycle. The derivatives of these coordinates should be single-valued functions. It is convenient to consider their exterior derivatives which can be expressed as linear combinations of the holomorphic and anti-holomorphic 1-forms and exact parts,

$$dX^0 = \sum_{i=1}^{g} \left(\lambda_i \omega_i + \bar{\lambda}_i \bar{\omega}_i\right) + \text{exact} \quad , \quad dX^9 = \sum_{i=1}^{g} \left(\gamma_i \omega_i + \bar{\gamma}_i \bar{\omega}_i\right) + \text{exact} \tag{6}$$

Then, we require

$$\oint_{a_i} dX^0 = \beta n_i + \sqrt{2}\pi R i p_i \quad , \quad \oint_{b_i} dX^0 = \beta m_i + \sqrt{2}\pi R i q_i \tag{7}$$

$$\oint_{a_i} dX^9 = \sqrt{2}\pi R p_i \quad , \quad \oint_{b_i} dX^9 = \sqrt{2}\pi R q_i \tag{8}$$

With (5), we use these equations to solve for the constants in (6). With the formula

$$\int \omega_i \bar{\omega}_j = \sum_{k=1}^{g} \left(\oint_{a_k} \omega_i \oint_{b_k} \bar{\omega}_j - \oint_{b_k} \omega_i \oint_{a_k} \bar{\omega}_j \right) = -2i \left(\Omega_2\right)_{ij} \tag{9}$$

we compute the part of the string action which contains the winding integers,

$$\begin{aligned} S = \frac{\beta^2}{4\pi\alpha'} \left(n\Omega^\dagger - m\right) \Omega_2^{-1} \left(\Omega n - m\right) + 2\pi i \frac{\sqrt{2}\beta R}{4\pi\alpha'} \frac{1}{2} \Big[\left(p\Omega^\dagger - q\right) \Omega_2^{-1} \left(\Omega n - m\right) \\ + \left(n\Omega^\dagger - m\right) \Omega_2^{-1} \left(\Omega p - q\right) \Big] + \ldots \end{aligned} \tag{10}$$

Note that the integers p_i and q_i appear linearly in a purely imaginary term in the action. Furthermore, since they come from the compactification of the light cone, this is the only place that they will appear in the string path integral (unlike m_i and n_i which should appear in the weights of the sum over spin structures). When the action is exponentiated and summed over p_i and q_i, the result will be periodic Dirac delta functions. It can be shown that these delta functions impose a linear constraint

on the period matrix of the worldsheet. Thus, with the appropriate Jacobian factor, the net effect is to insert into the path integral measure the following expression,

$$\sum_{mnrs} e^{-\frac{\beta^2}{4\pi\alpha'}\left(n\Omega^\dagger - m\right)\Omega_2^{-1}(\Omega n - m)}\nu^{2g}\left|\det \Omega_2\right| \prod_{j=1}^{g} \delta\left(\sum_{i=1}^{g}\left(n_i + i\nu r_i\right)\Omega_{ij} - \left(m_j + i\nu s_j\right)\right) \tag{11}$$

where $\nu = 4\pi\alpha'/\sqrt{2}\beta R$ is a fixed constant. Consequently, the integration over metrics in the string path integral is restricted to those for which the period matrix obeys the constraint

$$\sum_{i=1}^{g}\left(n_i + i\nu r_i\right)\Omega_{ij} - \left(m_j + i\nu s_j\right) = 0 \tag{12}$$

for all combinations of the $4g$ integers m_i, n_i, r_i, s_i such that Ω is in a fundamental domain.

Since the columns of the period matrix are linearly independent vectors, these are g independent complex constraints on the moduli space of Σ_g. Thus its complex dimension $3g-3$ is reduced to $2g-3$ and there is further discrete data contained in the integers. One would expect that, when the compactifications are removed, either $\beta \to \infty$ or $R \to \infty$, the discrete data assembles itself to a "continuum limit" which restores the complex dimension of moduli space.

It is interesting to ask whether the Riemann surfaces with the constraint (12) can be classified in a sensible way. The answer to this question is yes, a Riemann surface obeys the constraint (12) if and only if it is a branched cover of the torus, T^2, with Teichmüller parameter $i\nu$. This is established through the

Theorem: Σ_g is a branched cover of T^2 if and only if the period matrix obeys (12), for some choice of integers m_i, n_i, r_i and s_i.

Proof: The generators of the first homology group of T^2 are two closed loops (α, β) which span the vector space $H_1(T^2, \mathbb{C})$. The dual vector space, the first cohomology group, $H^1(T^2, \mathbb{C})$ is spanned by the basis of holomorphic and anti-holomorphic differentials γ and $\bar{\gamma}$. They can be normalized as,

$$\oint_\alpha \gamma = 1 \quad , \quad \oint_\beta \gamma = i\nu \quad \text{and} \quad \oint_\alpha \bar{\gamma} = 1 \quad , \quad \oint_\beta \bar{\gamma} = -i\nu \tag{13}$$

The Riemann surface Σ_g is a branched cover of T^2 if there exists a continuous, onto, holomorphic map f, such that

$$\Sigma_g \xrightarrow{f} T^2 \tag{14}$$

The map f takes closed loops on Σ_g to closed loops on T^2. In particular, the generators (4) must map as

$$(a_i, b_j) \xrightarrow{f} (n_i\alpha + r_i\beta, m_j\alpha + s_j\beta) \tag{15}$$

for some integers m_i, n_i, r_i, s_i. This gives a mapping between the vector spaces $H_1(\Sigma_g, \mathbb{C})$ and $H_1(T^2, \mathbb{C})$. A mapping of vector spaces induces a pull-back on the dual vector spaces

$$H^1(T^2, \mathbb{C}) \xrightarrow{f^*} H^1(\Sigma_g, \mathbb{C}) \tag{16}$$

defined by its action on the basis,

$$\begin{aligned} a_i \circ f^*(\gamma) &= \oint_{a_i} f^*(\gamma) \equiv f(a_i) \circ \gamma = n_i \oint_\alpha \gamma + r_i \oint_\beta \gamma = n_i + i\nu r_i \\ b_j \circ f^*(\gamma) &= \oint_{b_j} f^*(\gamma) \equiv f(b_j) \circ \gamma = m_j \oint_\alpha \gamma + s_j \oint_\beta \gamma = m_j + i\nu s_j \end{aligned} \tag{17}$$

Consider the particular elements of $H_1(\Sigma_g, \mathbb{C})$,

$$c_j = \sum_{i=1}^{g} a_i \Omega_{ij} - b_j \tag{18}$$

It can be checked from the definition of the period matrix (4) (and the fact that it is symmetric) that any holomorphic differential, η, on Σ_g has the property

$$c_j \circ \eta = \sum_{i=1}^{g} \Omega_{ij} \oint_{a_i} \eta - \oint_{b_j} \eta = 0 \tag{19}$$

The holomorphic nature of the mapping, f, guarantees that $f^*(\gamma)$ is a holomorphic differential on Σ_g. Then it follows that

$$0 = c_j \circ f^*(\gamma) = f(c_j) \circ \gamma = \sum_{i=1}^{g} (n_i + i\nu r_i)\Omega_{ij} - (m_j + i\nu s_j) \tag{20}$$

which is the constraint on the period matrix in eq.(12).

To prove the converse, we must show that if the constraint (12) is satisfied, then a covering map, f, exists. We will demonstrate this by explicit construction. Consider the line integral of a linear combination of the holomorphic differentials on Σ_g,

$$z(P) = \int_{P_0}^{P} \sum_{k=1}^{g} \lambda_k \omega_k \tag{21}$$

with P_0 a fixed base-point. We wish to choose the coefficients λ_k such that this integral defines a map from points $P \in \Sigma_g$ to the torus $z(P) \in T^2$ whose holomorphic coordinates are the complex numbers $z(P)$ with the identification $z \sim z + p + i\nu q$ where p and q are integers. The integral depends on the path of integration. If the path is changed by a combination of homology cycles of Σ_g, $k_i a_i + l_i b_i$, the integral on the right-hand side of (21) changes by

$$\delta z = \sum_j \lambda_j \left(k_j + \sum_i \Omega_{ji} l_i \right)$$

λ_k must be chosen so that this change is commensurate with the periods of T^2. This can easily be done if Ω obeys (12). Then, with the choice $\lambda_i = n_i + i\nu r_i$,

$$\delta z = \sum_j (n_j + i\nu r_j)k_j + \sum_i (m_i + i\nu s_i)l_i = \text{integer} + i\nu \cdot \text{integer}$$

and we have constructed an explicit covering map, $z(P)$. **Q.E.D.**

As a concrete example, the constraint (12) can be solved explicitly for genus one. The leading contribution to the free energy of the finite temperature type II superstring (without DLCQ) is from the torus amplitude and is given by [11]

$$\frac{F}{V} = -\int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \sum_{mn} e^{-\frac{\beta^2|n\tau-m|^2}{4\pi\alpha'\tau_2}} \left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^5 \frac{1}{4\,|\eta(\tau)|^{24}} \Big[\left(\theta_2^4\bar\theta_2^4 + \theta_3^4\bar\theta_3^4 + \theta_4^4\bar\theta_4^4\right)(0,\tau) +$$
$$+ e^{i\pi(m+n)}\left(\theta_2^4\bar\theta_4^4 + \theta_4^4\bar\theta_2^4\right)(0,\tau) - e^{i\pi n}\left(\theta_2^4\bar\theta_3^4 + \theta_3^4\bar\theta_2^4\right)(0,\tau) - e^{i\pi m}\left(\theta_3^4\bar\theta_4^4 + \theta_4^4\bar\theta_3^4\right)(0,\tau)\Big] \quad (22)$$

where $\theta_k(0,\tau)$ are Jacobi theta functions and $\eta(\tau)$ is the Dedekind eta-function. $\mathcal{F}$ is the fundamental domain of the torus,

$$\mathcal{F} \;\equiv\; \left\{\tau = \tau_1 + i\tau_2 \,\middle|\, -\frac{1}{2} < \tau_1 \le \frac{1}{2};\; |\tau| \ge 1; \tau_2 > 0\right\} \quad (23)$$

The modification of this formula by the null compactification can be found using (11),

$$\frac{F}{V} = -\sum_{\tau\in\mathcal{F}} \frac{\nu^2}{m^2+\nu^2 n^2} e^{-\frac{\beta^2|n\tau-m|^2}{4\pi\alpha'\tau_2}} \left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^5 \frac{1}{4\,|\eta(\tau)|^{24}} \Big[\left(\theta_2^4\bar\theta_2^4 + \theta_3^4\bar\theta_3^4 + \theta_4^4\bar\theta_4^4\right)(0,\tau) +$$
$$+ e^{i\pi(m+n)}\left(\theta_2^4\bar\theta_4^4 + \theta_4^4\bar\theta_2^4\right)(0,\tau) - e^{i\pi n}\left(\theta_2^4\bar\theta_3^4 + \theta_3^4\bar\theta_2^4\right)(0,\tau) - e^{i\pi m}\left(\theta_3^4\bar\theta_4^4 + \theta_4^4\bar\theta_3^4\right)(0,\tau)\Big] \quad (24)$$

where the the solution of (12) yields the discrete Teichmüller parameter,

$$\tau = \frac{m + i\nu s}{n + i\nu r}$$

and one should sum over the integers so that τ is in the fundamental domain, $\mathcal{F}$.

In ref.[10] it was shown that this formula can also be obtained by operator methods by finding the spectrum of the non-interacting type II superstring in DLCQ and explicitly computing the thermodynamic free energy by summing over energy states. This provides a strong check of the path integral technique for DLCQ we have used in the present paper.

Modular transformations and identities for theta functions can be used to rewrite (24) as the Hecke operator [12] acting on the partition function of a superconformal field theory, with torus worldsheet and target space R^8:

$$\frac{F}{V} = -\frac{1}{\sqrt{2}\pi R\beta}\mathcal{H}[e^{-\beta/\sqrt{2}R}] * \left[\left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^4 \frac{1}{|\eta(\tau)|^{24}}\,|\theta_4(0,\tau)|^8\right]_{\tau=-1/i\nu} \quad (25)$$

A modular transform can be used to write this as

$$\frac{F}{V} = -\frac{1}{\sqrt{2}\pi R\beta}\mathcal{H}[e^{-\beta/\sqrt{2}R}] * \left[\left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^4 \frac{1}{|\eta(\tau)|^{24}} |\theta_2(0,\tau)|^8\right]_{\tau=i\nu} \tag{26}$$

The factor in front is the ratio of volumes of R^8 and $R^9 \times S^1$ with compactified light cone. The action of $\mathcal{H}[p]$ on a function $\phi(\tau,\bar{\tau})$ is defined by

$$\mathcal{H}[p] * \phi(\tau,\bar{\tau}) = \sum_{N=0}^{\infty} p^N \sum_{\substack{kr=N,\ r\ \text{odd} \\ s \bmod k}} \frac{1}{N}\phi\left(\frac{r\tau+s}{k}, \frac{r\bar{\tau}+s}{k}\right) \tag{27}$$

In [10], this formula was shown to arise in the $g_s \to 0$ limit of matrix string theory at finite temperature. In this limit, matrix string theory reduces to a theory of eigenvalues of matrices which naturally live on covers of the torus. Generally, these should be branched covers. In the leading order in g_s only the covers without branch points contribute. It was shown that the thermodynamic free energy of the matrix model arising from summing over unbranched covers is identical to (26). The combinatorics of enumerating them is elegantly accounted for by the Hecke operator.

In matrix string theory the limit $g_s \to 0$ corresponds to large gauge coupling. This limit projects the theory onto the zeros of the superpotential. These zeros occur when all of the matrices are simultaneously diagonalizable. This is why the matrix model reduces to a theory of eigenvalues [3]. Beyond the leading order in strong coupling, the quadratic fluctuations of off-diagonal parts of the matrices can also be analyzed [6]. It is found that the fluctuation determinants are ultra-local operators and almost cancel due to supersymmetry. Arguments are given that, when carefully treated, the gauge field sector produces the power of the string coupling constant g_s^{2g-2} accompanying the branch covers of genus g (for a discussion of gauge fields, see refs.[6, 13, 14]). The same argument would apply to an analysis of the matrix theory at finite temperature. Even though the supersymmetry is broken by temperature boundary conditions, since the fluctuations of non-diagonal fields are governed by ultra-local operators, the same supersymmetric cancellations should occur. The remaining theory of diagonal matrices is identical to the finite temperature Green-Schwarz superstring with worldsheets which are branched covers of the torus. In this paper we have shown that this is exactly what is realized in the DLCQ of string theory at finite temperature.

What must be done to demonstrate that this limit of matrix string theory describes the perturbative type-II superstring? It must be shown that the integration measures over the worldsheets in the string theory and over the collective coordinates of the branched covers, which can be regarded as instantons in matrix theory, are identical.

The authors thank M. Goresky, J. Harvey, I. Klebanov, J. Kollár, Y. Matsuo, D. Morrison, E. Verlinde and K. Zarembo for helpful conversations.

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